Parameter Optimization—An Aid to Thermal Protection Design

S. D. WILLIAMS*
Lockheed Electronics Company Inc., Houston Aerospace Systems Division, Houston, Texas
AND
D. M. CURRY†
NASA Manned Spacecraft Center, Houston, Texas

This paper documents the numerical parameter optimization techniques used to determine an optimum weight for a thermal protection system. Subject to temperature constraints imposed at various locations in the system, the most impressive optimization technique was an adaptive creep search followed by a pattern search. The thermodynamic problem was solved using an implicit method for reasons of numerical stability and computational speed. Numerical optimization techniques were shown to be beneficial in the design of a thermal protection system. Numerical results were obtained showing the effects of temperature constraints, materials, and material arrangement on the weight of a thermal protection system.

Introduction

THE design and development of a minimum-weight thermal protection system (TPS) for the space shuttle is desirable since this would allow the payload capability to be maximized. However, this requires optimum material selection, arrangement, and thickness. A typical solution to this problem is a parametric study and the generation of correlations: a thermal model is formulated and programed for a digital computer; then a series of simulations is performed by varying the insulation thicknesses until the desired temperature constraint is achieved. For a two-layer slab system, one constraint, this iterative approach is quite efficient. Interpolation of previous results can be used to predict the required thickness.

The application of this simple iterative technique to a multilayer insulative TPS with air gaps, two or more constraints, however, can be quite tedious and time-consuming. Therefore, a study was made to develop a method and computer program to determine the feasibility of using existing numerical optimization techniques to assist in the design of a minimum-weight configuration for a typical space shuttle TPS independent of structural considerations.

Several different numerical parameter optimization techniques were investigated, ranging from simple one-dimensional searches to more sophisticated techniques such as Davidon's. An implicit thermal model was used in conjunction with the optimization techniques. Temperature constraints were imposed at the back surface of the insulation materials, and manufacturing constraints were imposed on material thickness. The most efficient optimization method used was an adaptive creep-pattern search. This method does not require the evaluation of derivatives to obtain the minimum and has been successfully used to achieve convergence in one computer run.

Multivariable Optimization

The general nonlinear multivariable optimization problem is concerned with finding the extremum of a performance index of the form

$$\tilde{\phi} = \tilde{\phi}(x) \tag{1}$$

where $\mathbf{x}$ is an n dimensional vector with components x_i, subject to an $\mathbf{m}$ vector of constraints

$$\psi_j = \psi_j(x) \tag{2}$$

The x_i are independent variables, control parameters, whose values are to be determined such that Eq. (1) is an extremum subject to the constraints in Eq. (2). If the constraints are applied directly to the independent variable

$$x_i^L \leq x_i \leq x_i^H, \qquad i = 1,2,..,m \tag{3}$$

then a region of control space is defined in which the solution must lie. Problems involving equality constraints can be treated as unconstrained problems by replacing the actual performance index with a penalized performance index ϕ, where

$$\phi = \tilde{\phi} + W_j\psi_j^2 \tag{4}$$

The repeated subscript indicates the usual tensor summation, and the W_j are positive weighting constraints. It can be shown that if the W_j are sufficiently large in magnitude, minimization of Eq. (1) subject to the constraints of Eq. (2) is equivalent to the minimization of the unconstrained performance index defined by Eq. (4). This approach permits search techniques for finding unconstrained minima to be applied in the solution of constrained minima.

A variety of numerical procedures have been developed to solve parameter optimization problems. Most of the searches which proved effective were based on the reduction of the multidimensional control space to a succession of steadily improving searches along a vector. Thus, the search technique can be thought of as a one-dimensional search technique. The numerical search for the minimum of $\tilde{\phi}$ can be carried out in a local region by most methods, but none can guarantee the global minimum. The object of these numerical methods is to isolate the minimum performance index as rapidly as possible, often with little or no previous knowledge of the characteristics of the response surfaces. A measure of the effectiveness of the various search techniques used in this report is the number of evaluations required to locate the minimum.

Williams[1] discusses several numerical optimization techniques which have been used to solve for a minimum weight TPS. For the reader interested in more detailed information on parameter optimization than is contained in Ref. 1, the excellent survey paper by Spang[2] is recommended. Discussions on the Golden Section, which is a one-dimensional search, are given by Wilde[3] and Kiefer.[4] The original development on the adaptive creep and pattern searches was done by Hooke and Jeeves[5] and by Wood.[6] The Davidon method was first reported by Davidon[7] and was later modified by Fletcher and Powell.[8] A newer method that is supposed to

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* Scientific Programing Specialist, Engineering Applications Department.

† Aerospace Technologist, Thermal Technology Branch, Structures and Mechanics Division.

provide additional advantages has been developed by Davidon.[9] Although the Newton–Raphson and quadratic techniques have been in use for several years, the papers by Orden[10] and by Goldfeld, Quandt, and Trotter[11] give excellent justification for the use of the quadratic method. The method of steepest descent, which has been widely used, is discussed in detail by Curry.[12] The program description of all of these techniques can be found in Ref. 13.

Adaptive Creep

The adaptive creep search is a form of small-scale sectioning which perturbs the control variable α_i by a small amount $\Delta\alpha_i$ in the descending direction instead of locating the minimum on each section parallel to the axes. If the surface is relatively uncoupled, this method will find the minimum in an efficient manner.

The search starts with a perturbation $\Delta\alpha_i$ in one of the control variables α_i. The perturbation is first made to the right, increasing the value of α_i. If this fails to decrease the performance index, a step is made to the left. If neither perturbation makes an improvement in the performance, the variable retains its nominal value, and the $\Delta\alpha_i$ is reduced. If the performance index is reduced, then α_i is set to this value, and the $\Delta\alpha_i$ is increased. This process is continued until each control variable has been perturbed.

A two-dimensional illustration of the adaptive creep search is shown in Fig. 1. If the perturbation size is decreased by a factor n and increased m during the search process, then the perturbation in the ith control variable can be written as

$$\Delta\alpha_i = m^{\mu} n^{\nu} \Delta^{\circ}\alpha_i \tag{5}$$

where ν is the number of searches in which the perturbation has been unsuccessful, and μ is the number of searches in which a perturbation has been successful. $\Delta^{\circ}\alpha_i$ is the original perturbation step for the ith control variable.

While the adaptive creep may appear similar to a one-dimensional search, it is actually more sophisticated since each perturbation is scaled independently. The perturbation in each variable is adaptively determined on the basis of the performance index response contour behavior encountered. Since the search often approaches the minimum with perturbations of increasing magnitude, it has a tendency to overshoot the extremal. To compensate for this, the values for n and m in Eq. (5) are 10 and 2, respectively. In addition, a lower limit is imposed on each $\Delta\alpha_j$ beyond which it can no longer be decreased.

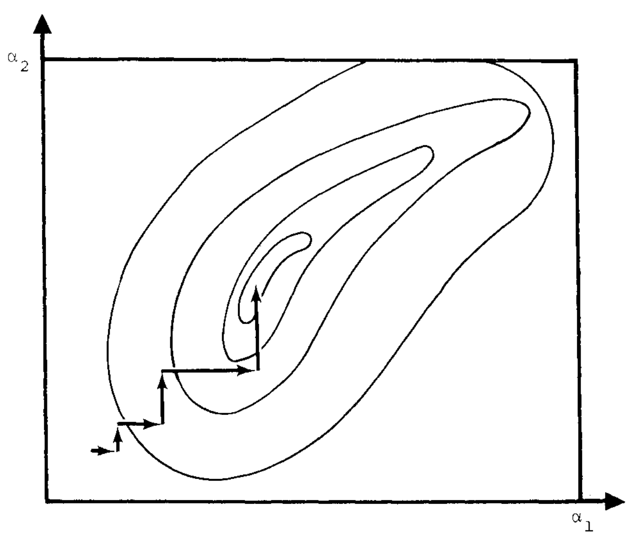

Fig. 1 Adaptive creep stepping to the minimum.

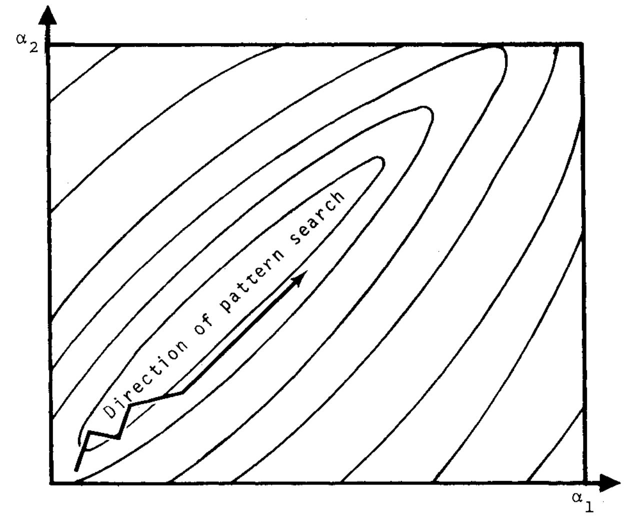

Fig. 2 Pattern search following another search technique.

Pattern Search

The pattern search explores a gross direction revealed by one of the previous searches. The search direction and magnitude are given by

$$\Delta\alpha_i = \mu(\alpha_i^{j+1} - \alpha_i^{j}) \tag{6}$$

where α_j^j is the value of the ith control variable at the end of the jth search. The algorithm consists of doubling μ until a degradation in performance occurs. The initial value of μ is 1. This type of search can prove very advantageous when two successive searches produce a small change in the performance index (Fig. 2).

Efficiency Study

Consideration is given in this study to the convergence characteristics of various optimization techniques applied to finding the minimum weight of a TPS subject to temperature constraints imposed at the backwall of a material. These temperature constraints are based upon the maximum temperature encountered at fixed locations during the trajectory. The problem can be stated mathematically: minimize the performance index ϕ where

$$\phi = \rho_i x_i, \qquad i = 1,2,..,n \tag{7}$$

subject to the constraints ψ_k,

$$\psi_k = T_j - T_k^*, \qquad k = 1,2,..,m \tag{8}$$

The repeated subscript indicates the usual tensor summation, $m \leq n$, ρ_i is the density for material i with thickness x_i, T_j is the temperature at the backwall of material j, and T_k^* is the desired temperature for the kth constraint. An additional set of constraints is imposed on the control.

$$x_{l\min} \leq x_l \leq x_{l\max}, \qquad l = 1,2,..,p \tag{9}$$

where $p \leq n$, and the minimum and maximum thicknesses are engineering or manufacturing limitations on the dimensions of the material.

A discussion of the thermal model is given in Ref. 14 and the existence of a minimum weight TPS is based upon the uniqueness of the solution to the heat equation. Thermal properties were taken from Refs 1, 15, and 16. The thermal conductivity and specific heat were temperature dependent for all materials.

Three materials (Dynaflex, TG-15000, and aluminum) were chosen to represent the TPS for this study. These materials are representative of the materials applicable to the space

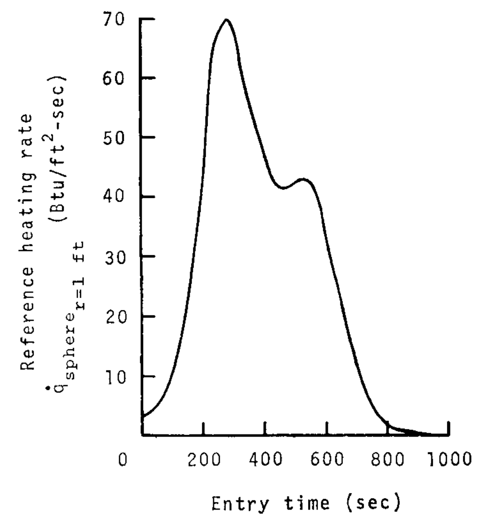

Fig. 3 Reference heating rate as a function of time for a 60° angle of attack during re-entry.

shuttle. The control variables used for the optimization were the thickness of the Dynaflex and TG-15000. A thin metallic skin was assumed to form the aerodynamic surface to which the Dynaflex was attached; however, this thin skin was ignored in the thermal model. The thickness of the aluminum was held to a constant value of 0.07 in. The maximum temperature at the backwall of the Dynaflex was constrained to 1660 ± 0.5°R, while the backwall of the aluminum was constrained to 660 ± 0.5°R. No temperature constraint was imposed on the backwall of the TG-15000.

Since the purpose of this effort is to evaluate optimization techniques, a trajectory time of 500 sec was used for the thermal model. The convective heating rate, given in Fig. 3, is a design heating rate for a shuttle re-entry at an angle of attack of 60°. The heating rate shown is for a 1-ft radius sphere. This rate was modified by a heating factor of 0.41 to provide surface temperatures typical of the leading edge and fuselage nose areas.

Three specific TPS models were considered (Fig. 4): 1) case 1—Dynaflex/TG-15000/aluminum, 2) case 2—Dynaflex/air gap/TG-15000/aluminum, and 3) case 3—Dynaflex/TG-15000/air gap/aluminum.

The emissivity for the air gap was 0.2 on both the front and back of the materials. An emissivity of 0.8 was used at the face of the Dynaflex for surface radiation to space.

Four optimization methods were chosen for this study: 1) an adaptive creep search followed by a pattern search, 2) a steepest descent search followed by a pattern search, 3) a Davidon search followed by a pattern search, and 4) a quadratic search followed by a pattern search.

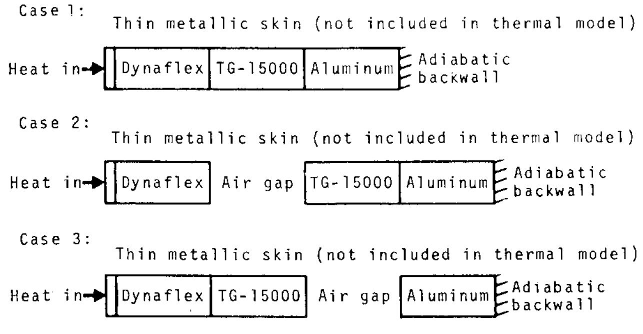

Fig. 4 The three different thermal protection system models used for optimization study.

A pattern search is used after a major search technique since it saves a large number of computations if it improves the performance, and costs one evaluation of the performance index if it fails. It should be pointed out that all of these methods attempt to satisfy the constraints first since this yields the greatest reduction in the performance index. After the constraints have been satisfied, the performance index is minimized within the constraint region. The final tolerance on each constraint was ±0.5°R. This tight tolerance was used to eliminate any variance between the minimums that might be caused by the different optimization techniques, i.e., Davidon, Steepest Descent, etc. For the models shown in Fig. 4, the minimum weight is achieved when the constraints are satisfied. Although a detailed study has not been performed on the sensitivity of these constraints on the efficiency of achieving convergence, experience with this program indicates that if a broader tolerance is specified, additional computer time will be required to achieve the minimum within the constraint region.

The most efficient optimization method used was the adaptive creep-pattern for all three TPS models studied. For case 1 (no air gap), the minimum weight was achieved in 11 cycles and 191 evaluations of the performance index. For case 2 (with the air gap between the Dynaflex and the TG-15000), the algorithm indicated convergence in eight cycles and 147 evaluations of the performance index. Actually, one of the constraints was not satisfied, but the solution was achieved for practical purposes. For case 3 (with the air gap between the TG-15000 and the aluminum), convergence was achieved in 10 cycles and 171 evaluations of the performance index.

No method was effective enough to be considered as an alternate method to the adaptive creep-pattern. This resulted from several factors: 1) the effect of the penalized temperature constraints tends to dominate the perturbations even though the performance index is linear; 2) the response surface is relatively uncoupled; 3) the second-order techniques encounter numerical difficulties which are not accounted for in the program; and 4) the second-order partial derivatives do not exist. A summary of the results for all cases is given in Table. 1

A cycle for the adaptive creep-pattern search allowed five adaptive creep searches followed by a pattern search. The ordering of the perturbations (i.e., which control variable was changed first) was determined on a random basis as opposed to natural order.

A plotting capability was added to the program to plot various parameters as functions of the cycle. Figure 5 demonstrates graphically the response of the different parameters during the progress of the optimization search. A study was made of these data to determine the most efficient number of adaptive creep searches to use during each cycle. Only case 3 was used for this study. The most efficient number was determined to be one adaptive creep search followed by a pattern search. An interesting occurrence was noted during this phase of the investigations: the number of evaluations for five adaptive creep searches per cycle was reduced from 171 to 136. This reduction resulted from the random selection of which variable was perturbed first. Even with this improvement, one adaptive creep search per cycle results in over a 20% reduction in computational time.

Effects of Temperature Constraints on Weight

The three cases shown in Fig. 4 were re-examined using a trajectory time of 1500 sec to determine the effects of temperature constraints on the weight of the TPS. In these studies the face of the Dynaflex was assumed to radiate to space, and no allowance was made to account for the atmospheric effects on radiation during re-entry or after touchdown.

Changing the aluminum backwall temperature while holding the Dynaflex backwall temperature constant produced a greater change in weight than changing the Dynaflex backwall

Table 1 Summary of optimization techniques for solving the three cases.

Case	Search	Number of cycles	Number of evaluations	Converged	Comments
1	Adaptive creep-pattern	11	191	Yes	
	Steepest descent-pattern	12	526	No	
	Davidon-pattern	5	381	Yes	
	Quadratic-pattern	3	67	No	$A^{-1} = 0$
2	Adaptive creep-pattern	8	147	Yes	[a]
	Steepest descent-pattern	9	370	No	
	Davidon-pattern	4	272	No	[b]
	Quadratic-pattern	2	43	No	$A^{-1} = 0$
3	Adaptive creep-pattern	10	171	Yes	
	Steepest descent-pattern	9	329	No	
	Davidon-pattern	2	158	No	[b]
	Quadratic-patttern	2	44	No	$A^{-1} = 0$

temperature while holding the aluminum backwall temperature constant. This effect can be seen in Figs. 6a and 6b. Of the three cases examined, case 2, which placed the air gap between the second and third materials (TG-15000 and aluminum), achieved the greatest weight saving. It is interesting to note that placing the air gap between the Dynaflex and the TG-15000 did not significantly reduce the weight from when no air gap at all was used.

Based on the specific configuration and materials investigated, several conclusions may be drawn: 1) A greater reduction in weight can be achieved by placing an air gap in a region of low temperature rather than in a region of high temperature; 2) allowing a higher temperature tolerance on the backwall of the TPS can be more beneficial in reducing weight than allowing the same tolerance on temperatures at other locations, such as the bond line; and 3) the location of an air gap is almost as important as the backwall temperature in optimizing the TPS weight.

Material Analysis

The space shuttle orbiter TPS will experience surface temperatures varying from 960° to 3460° R. Both metallic and nonmetallic materials are being considered as possible candidates for the TPS. The refractory metals and superalloys appear to be the nearest state-of-the-art materials available that can be considered reusable. However, these systems have inherent problems which must be solved before they can be considered operational.

Another class of materials, the nonmetallic insulators, are currently under development, and preliminary results indicate they offer a potentially better choice as a reusable TPS. Therefore, a nonmetallic TPS configuration was selected as the basic system for the weight analysis in this section. Four materials (Lockheed LI-1500, titanium plate or titanium honeycomb, TG-15000, and aluminum) were considered as forming the basis of this system.

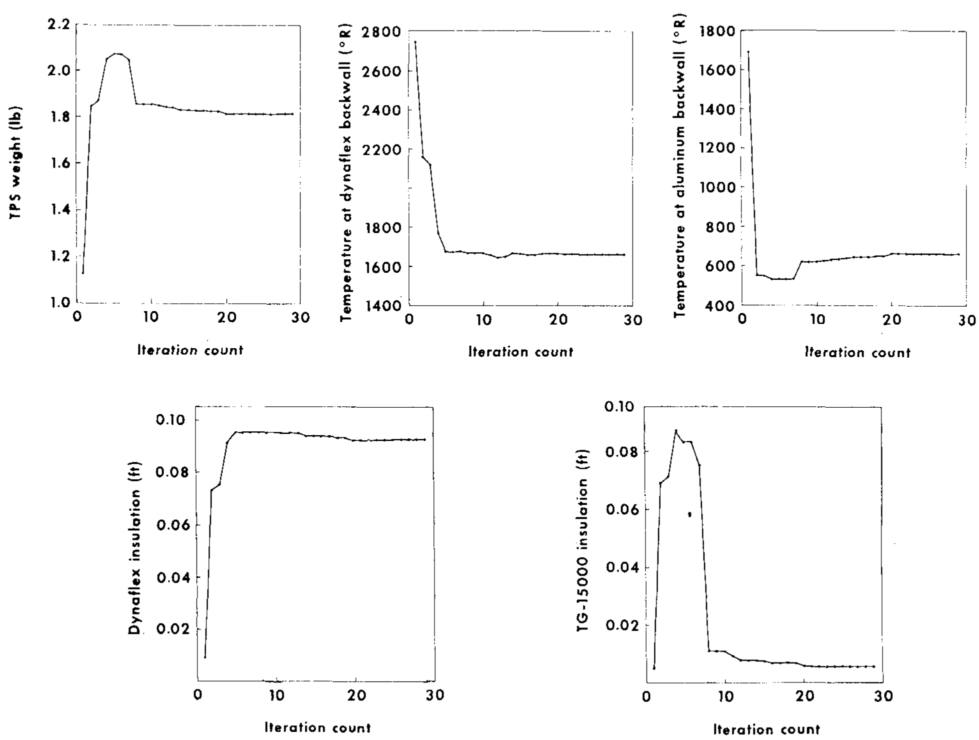

Fig. 5 Effects of the performance index, temperature constraints, and control variables as functions of iteration count for one adaptive creep search followed by a pattern search.

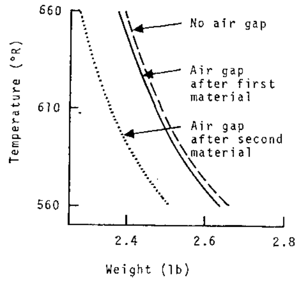

Fig. 6a Temperature change at the backwall of aluminum as a function of TPS weight. Dynaflex backwall temperature was held constant at 1660° R.

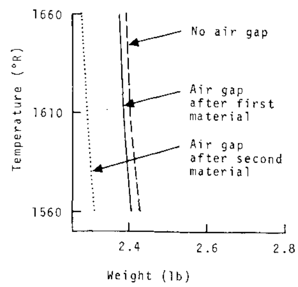

Fig. 6b Temperature change at the backwall of Dynaflex as a function of TPS weight. Aluminum backwall temperature was held constant at 660° R.

The surface material for each model studied was the LI-1500 insulator. The TPS consists of the LI–1500 bonded to either titanium subpanel (plate or honeycomb) attached to an aluminum support structure. For two of the models investigated, a low-density insulation, TG-15000, was placed behind the titanium subpanel. The various configurations and material combinations studied are not meant to imply that a specific shuttle TPS is preferable, but only to emphasize the generality of numerical optimization to TPS design.

The objective in each case was to determine the minimum insulation weight (LI-1500/TG-15000) of the TPS subject to material temperature and thickness (minimum gage) constraints. The weights presented are for the insulations only and do not include support posts, attachments, closeouts, or any load-bearing structure which would lead to be considered in the total TPS weight. For the results presented in this section, a heating rate history typical of the delta wing orbiter (1100 naut. mile) has been employed. A trajectory time of 2800 sec. with the heating rate shown in Fig. 7 was used to simulate entry conditions. This heating rate was modified by heating factors from 0.2 to 0.025 to represent realistic surface temperatures and heat loads. Only the results using a heating factor of 0.1 are shown since similar results were obtained for the range of conditions investigated.

To prevent degradation and loss of reuse capability, the bondline temperature between the LI-1500 and the titanium

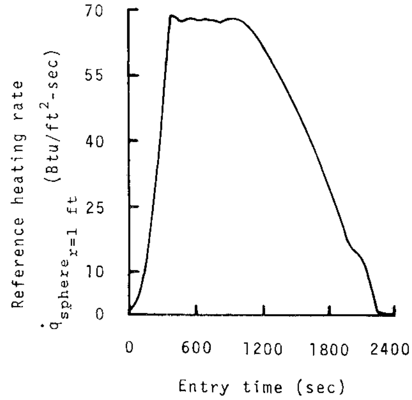

Fig. 7 Reference heating rate as a function of time for a delta wing orbiter (1100 naut mile crossrange).

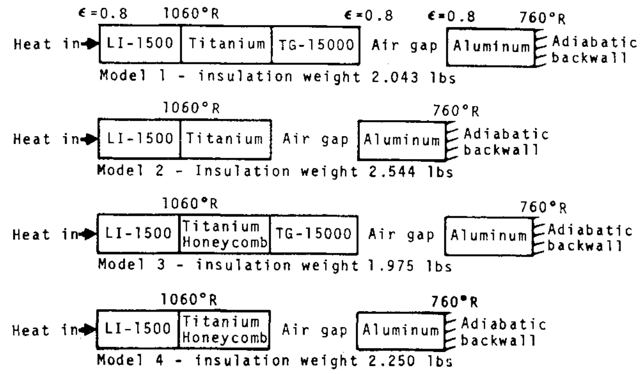

Fig. 8 The four TPS models considered for analysis.

subpanel was restricted to 1060° R, and the aluminum backwall was constrained to 760° R. For each model the thicknesses of the aluminum and titanium (panel or honeycomb) were held constant. An emissivity of 0.8 was used for the LI-1500 surface and on each side of the air gap. A uniform initial temperature of 560° R was used for all calculations.

Four different TPS models were considered using the materials shown in Fig. 8:

1) Model 1—LI-1500 bonded to titanium subpanel with TG-15000 behind the panel. An air gap was placed between the TG-15000 and aluminum.

2) Model 2—LI-1500 bonded to titanium. An air gap was placed between the titanium and aluminum.

3) Model 3—LI-1500 bonded to titanium honeycomb with TG-15000 behind the panel. An air gap was placed between the TG-15000 and aluminum

4) Model 4—LI-1500 bonded to titanium honeycomb. An air gap was placed between the honeycomb and aluminum.

Convergence was achieved for all models; however, each model will be discussed individually.

For model 1, convergence was achieved with a unit weight of 2.043 lb and both temperature constraints were satisfied; however, the assumed TG-15000 minimum material thickness (0.5 in.) was being approached. At higher heating factors, the TG-15000 insulation was always driven to the minimum, and the critical temperature constraint was on the backwall of the LI-1500 since the temperature of the aluminum was maintained well below its critical value of 760° R. For lower heating factors, both temperature constraints were satisfied and and the effect of TG-15000 became more dominant in reducing the TPS weight, primarily due to its heat capacitance and lower density.

For model 2, convergence was achieved with a unit weight of 2.544 lb, but only the critical temperature on the aluminum backwall was satisfied. The temperature at the LI-1500/titanium bondline was maintained well below 1060° R for all heating factors. Even though the only difference between this model and Model 1 is the elimination of the TG-15000, an increase in weight of 0.5 psf was experienced.

Model 3 convergence was achieved with a unit weight of 1.975 lb and both temperature constraints were satisfied. This unit insulation weight is slightly less than that of Model 1. The TG-15000 minimum gage thickness was reduced to 0.1 in. and the thickness varied from 0.233 in. for a heating factor of 0.2 to 0.314 in. for a heating factor of 0.025.

Model 4 convergence was achieved with a unit weight of 2.250 lb which is more than that of Models 1 and 3, but less than Model 2. For this case, the critical temperature was at the backwall of the aluminum while the temperature at the LI-1500/titanium interface was well below 1060° R.

A complete weight analysis of each system requires the addition of the unit weights for the load carrying structures, coating, adhesive bond, packing, supports, etc., to the optimized insulation weights. The unit weights for both subpanel configurations are comparable with a slight weight savings possibly using the honeycomb panel.

The addition of TG-15000 resulted in lower unit insulation weights for both systems. However, the effect of the TG-15000 was more significant in reducing the insulation weight for the titanium plate than for the honeycomb panel. This was expected as a result of the lower thermal conductivity of the honeycomb panel.

The results of each model were obtained with a single computer run for each heating factor. Once again, this emphasizes the advantages of using optimization techniques in TPS design.

Conclusions

Optimization techniques can be readily applied to solving the difficult and realistic problems associated with the design of a minimum-weight space shuttle thermal protection system. The TPS was subjected to both temperature and material-thickness constraints. Each problem examined required only one computer run to achieve meaningful results.

The most successful optimization technique was an adaptive creep method coupled with a pattern search, which used an implicit model to numerically solve the heat equation. However, if the initial perturbation step is too small, the same convergence problems encountered by the steepest-descent and quadratic methods may occur.

The effects of temperature constraints on the weight of a TPS indicated that allowing a larger temperature tolerance on the backwall of the TPS can be more beneficial in reducing weight than permitting the same tolerance at other locations. Likewise, it was shown that the placement of the air gap can significantly affect the TPS weight.

Four models were studied to determine the best minimum weight system. This study made no attempt to give a complete analysis, but was used to illustrate the capability and applicability of the program for TPS design. The TPS, utilizing a honeycomb panel, appeared to offer the most advantage from a unit weight viewpoint. No attempt was made to optimize the honeycomb core, but additional weight reduction may be achieved by doing this. While the addition of the soft insulation gave a reduction in total unit insulation weight, other factors such as packaging may cause this advantage to be diminished.

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